Particle Physics

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Im reading [Gri11]. I want to start with some general remarks on physics. First it is generally believed that there are only four forces needed to explain the universe, those are gravity, electromagnetic, the strong and weak forces. Gravity is extremely weak in the context of individual elementary particles. To a certain extent the theories of Newton deal with the forces of gravity and the theory of Maxwell deal with electromagnetism. If we assume that these forces dont interact, and we can calculate the total force on a body by summing them then all we need is a theory that describes the weak and strong forces. Unfortunatley the predictions of Newton and Maxwell at the microscopic level need adjustment (QM) and I dont know that there is no interaction.

Griffiths makes a helpful distinction, “Please observe the distinction here between a type of mechanics and a particular force law. The force law tells you what F is, in the case at hand; the mechanics tells you how to use F to determine the motion”. Thus for example Newtons three laws propound to apply for all forces in the universe, and explain how they interact, while his law of gravitation tells us how to calculate the exact force due to one interaction. It is interesting to pay attention to what is implied by the general set up vs what requires the input of the specific force laws.

Using this distinction as well as some other observations I want to clarify what each physical theory is trying to do, in particular those that are *attempting* to deal with fundamental particles.

- Classical Mechanics: The general theory is Newtons three laws, the specific force laws are things like Newtons law of universal gravitation.

- Electromagnetism: The theory is Maxwells equations the force law is given by

$$F = q(E + v \times B).$$

  These classical theories fit nicely into Griffiths regime.

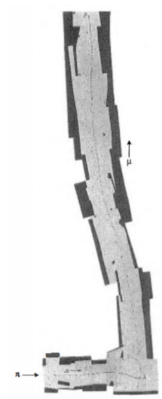
- Quantum Mechanics: Quantum mechanics (my understanding of it) does not have any force laws, it is not a theory of forces. It describes directly how a given state evolves. To some extent you could say that Schrodinger’s equation is the specific input while the setup with operators etc is the type of mechanics, but this feels forced.

- Special Relativity: .....
- General Relativity:
- Quantum Field Theory: Quantum field theory per se is the system that determines the mechanics, whilst it is the standard model that is the specific set of force laws. We will see I guess.

Other areas of physics dont deal with fundamental particles such as statistical mechanics, fluid dynamics, astrophysics, condensed matter etc. They of course use the information from these fundamental theories.

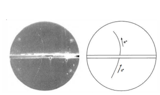
1 Experiments

The first question is what it even means to detect an elementary particle. You cant see them, you cant feel them. What macro-observable phenomena implies there existence. Griffiths tells us



> As it turns out, almost all of our experimental information comes from three sources: (I) scattering events. in which we fire one particle at another and record (for instance) the angle of deflection; (2) decays, in which a particle spontaneously disintegrates and we examine the debris; and (3) bound states. in which two or more particles stick together and we study the properties of the composite object.
>
> Needless to say, determining the interaction law from such indirect evidence is not a trivial task. Ordinarily the procedure is to guess a form for the interaction and compare the resulting theoretical predictions with the experimental data,

Most of the elementary particles we get access to come also in three forms, from cosmic rays, nuclear reactors and particle accelerators. Protons, neutrons and electrons are easy to get from ordinary matter by heating up hydrogen or metals. Particle accelerators are the newest tech, initially we only had cosmic rays. Particle accelerators can collide readily accessible electrons and protons, then using powerful magnets separate different smaller particles that can be collected and then fired at one another.



The detectors that then surround these interactions then work on the principle "when high-energy charged particles pass through matter they ionize atoms along their path". The ionized paths are then detectable (for instance in the oldest methods the ions are created in a cooled alcohol vapour and act as a catalyst for the vapour to condensate). Something to note is that "electrically neutral particles do not cause ionization, and they leave no tracks ... their paths have to be reconstructed by analyzing the tracks of the charged particles in the picture and invoking conservation of energy and momentum at each vertex". The curvature of the paths is directly related to the momentum and sign of the charge of the particles (electromagnetism), thus the paths tell you the momentum of the particles.

The story of the 20th century then is the story of the discovery of many new little particles. As more were discovered the theoretical frameworks changed and expanded, predicted new particles that were subsequently found. The existence of the electron was discoved (first) in 1897, it was done exactly as described above, someone heated up a metal (a cathode ray) that shot a beam of something, then by putting it through an electromagnetic field (and varying the field strength) he was able to determine the charge to mass ratio of the ejected particles. This ratio was very large, indicating large charge or tiny mass.

All said and done the theory now predicts 62 distinct elementary particles. 6 Leptons, 7 mediators and 6 (?) quarks, the quarks have three different colors and then every particle has an anti-particle dual, thus

$$2 \times (6 + 7 + 6 \times 3) = 62$$

The current theory suggests that thre cannot be any more particles under a given mass, and therefore if there are any more there is a large jump in the masses of the fundamental particles.

2 Lagrangian Field Theories

The formulation of physics that unifies the different paradigms (electromag, classical, QM) is Lagrangian. That is to say it is not immediately clear that there is a uniform language in which to state say the Maxwell equations and the Schrodinger equation.

For classical mecahincs [LL76] puts it as follows. A physical system is characterised by a function $L(x, \dot{x}, t)$ by the requiring the system moves between two points $(x_1, t_1) \to (x_2, t_2)$ in a way that extremises the integral,

$$S = \int_{t_1}^{t_2} L(x, \dot{x}, t) dt$$

This is equvalent (? certainly it is sufficient) to $x(t)$ satisfying the Euler-Lagrange equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} = \frac{\partial L}{\partial x_i}$$

(one equation for each generalised coordinate, for instance one particle in three dimensions requires three equations).

In a field theory we can do an analogous thing. Let $\varphi(x, y, z, t)$ be a vector field on $\mathbb{R}^4$ (say), for instance the magnetic field at a place and time. Because we want to be relativistic now too we will put space and time on an equal footing and simply denote

$$x^0 = t,\ x^1 = x,\ x^2 = y,\ x^3 = z$$

and we set

$$\partial_\mu \varphi = \frac{\partial \varphi}{\partial x^\mu}$$

Then the Lagrangian (or Lgrangian density) is a function of φ and the derivatives $\partial_\mu \varphi$, that lands in $\mathbb{R}$ (thus all vector quantities of interest will be resolved into scalars, for instance by taking dot products etc). Similarly we will then look for paths, that is x^μ, that minimize the integral over the Lagrangian, and the Euler-Lagrange equations become

$$\partial_\mu \frac{\partial L}{\partial \partial_\mu \varphi} = \frac{\partial L}{\partial \varphi}.$$

[Sch95, XIII.45] has a proper derivation and details for this, including the definition of the functional derivate. For instance the Dirac Lagrangian for a Spin 1/2 field is

$$L = \psi^\dagger \gamma^\mu \partial_\mu \psi - \psi^\dagger \psi,$$

where ψ is a "spinor field" in Griffiths language, which means simply a $\mathbb{C}^2$ vector field, γ^μ are some 2×2 "Dirac" matricies and we sum over repeated indecies. Notice that all the fields are sent to scalars by multiplying with their adjoints $\psi^\dagger$. The Euler lagrange equations then give you some differential equations on your field, in this case ψ must satisfy (summing over repeated indicies)

$$\partial_\mu \psi^\dagger \gamma^\mu + \psi^\dagger = 0.$$

> Up to this point, the Lagrangians we have considered might just as well describe classical fields as quantum ones. The passage from a classical field theory to the corresponding quantum field theory does not involve modification of the Lagrangian or the field equations, but rather a reinterpretation of the field variables.

Remark. In a certain sense then QM is already a field theory as the waves functions are naturally scalar functions, or if we add spin, that is $L^2(\mathbb{R}^4) \otimes \mathbb{C}^n$ then they become $\mathbb{C}^n$ vector fields. Indeed the (some cases of the?) Schrodinger equation can be derived from a certain Lagrangian formulation just as we described for the fields above, this is done in [Sch95, XIII.46] for

> We see then that the quantized field theory developed thus far in this section is equivalent to the Schrödinger equation for several non-interacting particles, provided that only the symmetric solutions are retained in the latter case. It can be shown that the two theories are completely equivalent even if interactions between particles are taken into account.

Or indeed [Fol08] and [PS19] perform this process for the Klein-Gordan equations which are just the relativistic Schrodinger equations.

Remark. What is the relationship between x and $\dot{x}$? A priori it is nothing. There is a relationship however between $x(t)$ and $\dot{x}(t)$, that is the position alone does nothing to tell you about the velocity, only the actual *trajectory* of the particle does that.

In this sense the Lagrangian or the Lagrangian density is a functional in the actual trajectories of x and $\dot{x}$. Also note that the relation between the two is simply that the Lagrangian density is a function over all space-time while the Lagrangian is just a function of time, after integrating out the space variables.

Remark. (Gauge Thoery) I really have come to hate this word. Griffiths puts it quite simply however. Before we quantize we can look at the Lagrangian. We can do one of two things, either observe an invariance under a substitution (or group action more generally) or we can try to enforce an invariance. Both can be more or less trivial depending on the situtaion. In practice what happens seems to be that a Lagrangian (classically) will have a "global" invariance, for instance

$$e^{i\theta}.(\psi^\dagger\psi) \;\; := \;\; (e^{i\theta}\theta)^\dagger(e^{i\theta}\theta) = \psi^\dagger\psi$$

this is global because our group action does not depend on the point x. One could ask what happens when we let θ depend on x. Obviously the answer is that it depends on *how* θ depends on x, but the point is that there exist (smooth) functions $\theta(x)$ such that for a Lagrangian globally invariant under this action it will not be preserved at every point x. This can be manually fixed by basically just adding in the difference (obviously not always this simple). This idea of *making* global symetries apply locally has been fruitful. For instance the Dirac Lagrangian above has a gloabl $e^{i\theta} \in U(1)$ symmetry / invariance when we force it to also be local the Lagrangian must change to be

$$\psi^\dagger\gamma^\mu\partial_\mu\psi - \psi^\dagger\psi \to \psi^\dagger\gamma^\mu\partial_\mu\psi - \psi^\dagger\psi - F^{\mu\nu}F_{\mu\nu} - \psi^\dagger\gamma^\mu\psi A_\mu$$

again summing over the repeated μ, ν's. Here $F_{\mu\nu} \;\; := \;\; \partial^\mu A^\nu - \partial^\nu A^\mu$ (covariant derivative) and A_μ is a vector field (we are summing over the components, so the Lagrangian is still a scalar) that transforms according to the rule

$$e^{i\theta(x)}.A_\mu = A_\mu + \partial_\mu\theta(x).$$

The point of this is that forcing Lagrangian to be locally "gauge invariant" necessitates the introduction of new fields, the A_μ. When we quantize these fields become particles, and so we have predicted that if the mechanics is locally gauge invariant then some particle must exist.

This was a $U(1)$ action on the Lagrangian (or some vector space that contains it, say $\mathbb{C}[L]$). Yang-Mills theory produced from a global $SU(2)$ invariant Lagrangian, for two spin half particles, and did the same process as above producing a locally invariant Lagrangian. The difficulty was dealing with the fact that $SU(2)$ was not abelian. Griffiths says that "the hard work is over: extending non-Abelian gauge theory to higher symmetry groups is a straightforward procedure, once the Yang-Mills model is on the table".

Remark. (Non-flat case) I guess merging QFT and GR is still an open problem, but one can see already that a lot of this was built on the idea of manifold theory already, or at least has a natural generalisation to it. We still have integrals vector fields, derivatives, covariant derivatives and even local group representations (principle bundles).

Remark. Ill just slap here that QCD is quantum chromo dynamics and QED is quantum electro dynamics. Because this theory deals with multiple forces (strong, weak, electromag) there are sort of sub theories that deal with just one (or two) that were developed more or less completely before the others. Namely QED was first worked out because QCD needed Yang-Mills theory first to make it work. These are all of the same form and indeed merge into the standard model.

3 Quantisation

Next we need to know how to quantise a field theoretic Lagrangian. There are multiple ways apparently, we will see how far we get, as this is not in Griffiths we will be consulting [PS19], [Ryd92] and [Fol08]. The processes in the physics texts are borderline incoherent, so Im drawing mostly from Folland. Folland claims that it makes sense for free fields and that fields with interactions are essentially not yet understood how to make what the physicists want to happen happen (it will be clearer later, but essentially the physicists say let blah be a thing such that bleh, and the point is that it is either known that no such blah exists or it is not known that one exists).

3.1 Axiomatic Answer

Berfore reading the exposition in Folland I found the physics answer essentially inpenetrable. So I will present this in the way that made it make sense to me. Following [Fol08, 5.5] "In the late 1950s Wightman and Garding formulated a list of basic properties that any physically reasonable and mathematically well defined quantum field theory should have". I will be ignoring gratuitous details.

First the theory consits of a few objects:

- a Hilbert space $\mathscr{F}$,
- a collection, $\varphi_1, ..., \varphi_N$, of $\mathscr{F}$ operator valued distributions on $\mathbb{R}^4$ (recall a *distribution* is in the functional analysis sense a map from $\mathcal{S}(\mathbb{R}^4)$, the Schwartz functions on $\mathbb{R}^4$, to in this case the set of operators),
- a unitary representation of $\mathbb{R}^4 \ltimes \mathrm{SL}_2(\mathbb{C})$ on $\mathscr{F}$ (this group is the universal cover of the set of Minkovski space time isometries),
- a representation of $\mathrm{SL}_2(\mathbb{C})$ on $\mathbb{C}^N$.

The φ_i's are our quantum fields on space time and the represenations capture the symmetries of space time. In the physics literature they dont care about distributions, thus we are supposed to really think of the quantum fields as just operator valued fields on $\mathbb{R}^4$, but mathematically we require distributions to make this work. We moreover require that the setup has some properties

- A bunch of technical things dealing with the fact that operators are only defined on dense subsets and we want them to be nice, even though they are unbounded.

- There should exist a "vacumme state" that is some element $\Omega \in \mathscr{F}$ that is stable under the unitary action on this Hilbert space (its stabiliser is the whole group) and moreover this state should be unique up to multiplication by $e^{i\theta}$.
- The fields operators acting on the vacumme state should give a Hilbert space basis of $\mathscr{F}$
- The fields operators should intertwine with the unitary operators of the representation on $\mathscr{F}$
- (Under some assumptions on the light cone points etc) the operators $\varphi_i(f), \varphi_j(g)$ should either commute or anticommute, as well as $\varphi_i(f)$ with $\varphi_j^\dagger(g)$.

"The bad news is that it has turned out to be a remarkably difficult task to construct examples of field theories that satisfy the Wightman axioms and have nontrivial interactions. Attempts to produce such examples in physical space-time R4 have yet to succeed". But at least we have a typing that makes sense for the free fields and a clear goal for what we want the whole thing to look like, even if it requires some technical generalisations.

Remark. This does nothing to tell us *how* to quantise a field, however it does tell us *what that even means*, that is it tells us what we want the output of our quantising process to be.

3.2 Physics Answer

So with that in mind lets go through what say [PS19] says about quantising the Klein-Gordan field (the presentation in [Ryd92] is very similar). Classically you start with a single scalar field φ and a Lagrangian density

$$L = (\partial_\mu \varphi)^2 - \varphi^2$$

Then "To quantize the theory, we follow the same procedure as for any other dynamical system: We promote φ and π to operators, and impose suitable commutation relations". Looking at the commutation relations in more classical theories and "replacing kronecker deltas with dirac deltas" results in imposing the following commutation relations on the "promoted" operators

$$[\varphi(x), \pi(x)] = i\delta(x - y),$$

and also that φ and π commute with themselves (at all points). This makes it clear that really when they say promote to an operator they mean a *field of operators*, while on the right hand side we dont have an operator so they are implicitly multiplying by the identity operator. Notice that they havent told us anything about what the "promoted" things actually are they just sort of say let there be two operators that commute in this way.

The next step is to try and expand the ansatzed fields in terms of Fourier series "In analogy with (quantum case) we write"

$$\varphi(x) = \int d^3p \quad (a_p + a_{-p}^\dagger)e^{ip\cdot x}, \quad \pi(x) = \int d^3p \quad (a_p - a_{-p}^\dagger)e^{ip\cdot x}$$

This step is the step that is supposed to relate it to the original φ, the reason is that their logic is that in QM if they replaced φ and π with the harmonic oscillator then they would be written basically as the integrands of the integrals above, but then they want to do this everywhere so they take an integral over all space.

3.3 Another Answer

Folland again comes to the rescue with a tidied up version of what they were trying to say above. Following [Fol08, 5] again we will deal with the Klein-Gordan equation. So consider a scalar field φ on (a box in) $\mathbb{R}^4$, then the Laplacian has an orthonormal basis of eigen functions $\{f_j\}$ (discrete spectrum if we restrict to a box). Expand the field in this basis

$$\varphi(t,x) = \sum q_j(t) f_j(x)$$

for some real valued functions q_j then Folland claims that "φ satisfies the Klein-Gordan equation iff its expansion satisfies"

$$q_j''(t) + \omega_j^2 q_j = 0, \quad \omega_j^2 = \lambda_j^2 + 1$$

for some ω_j and where λ_j are the eigenvalues for eigen function f_j. These are apparently the the equation for a classical Harmonic oscillator. Recall from QM that the Hamiltonian for K one dimensional oscillators can be written as

$$H = \sum \omega_j \left(A_j A_j^\dagger - \frac{1}{2} \right)$$

or by adding on a constant (which preserves the solutions / dynamics) simply

$$H = \sum \omega_j A_j A_j^\dagger$$

This aviods convergence issues latter. Now the key point is that because we have a *field* we cant consider individual particles yet and we therefore consider an *infinite number of oscillators.* For infinitely many particles we then need to ask what are these operators even acting on. The answer is the Fock space, which is just the complete (in a metric sense) symmetric tensor algebra over some hilbert space, in this case the relevant Hilbert space is some infinite dimensional seperable one (theyre all iso), we denote this space $\mathscr{F}$. Finally the position operator in QM has a relation to these A operators, so called ladder operators, given by

$$X_j = \frac{1}{\sqrt{\omega_j}} (A_j + A_j^\dagger)$$

We then construct the quantum field associated to φ as

$$\Phi(x) = \sum f_j(x) X_j$$

The time dependence can be restored by moving from the Schrodinger picture to the Heisenberg picture. Notice that what we have now is a field of operators. The problem is that this series almost never converges, Folland calculates its norm on the vacumme state and its is infinite. To fix this you introduce the distributional picture.

> We started from the idea of a classical field, that is, a function φ on (some region in) space-time whose value at a point x is an observable quantity — say, a force, a velocity, a temperature — that can be determined by measurements performed at x. One would expect quantization to yield a function Φ on space-time whose value at x is the quantum observable — i.e., self-adjoint operator — corresponding to the classical observable. What we seem to have ended up with, however, is rather different. The values of the quantum fields we have constructed are self-adjoint (or, more loosely, Hermitian) only when the quanta of the field are their own antiparticles; in other cases the values of the fields cannot directly represent observable quantities.

Remark. Notice that the quantised field has nothing to do with the original *field*, it is the Lagrangian itself that we are quantising, or rather the soltions to that Lagrangian. Thus the Φ we wrote down does not use the q_j coefficients from the original field in any way. The fields *happen* to have relations in some limits but this is not by virtue of the construction but rather because they both satisfy the DE.

Remark. This is not a repeatable process for other situations a priori (its not a posteriori either). Each classical situation seems to be quantised in some what ad hoc ways. The axiomatics makes it at least clear what they are aiming at. Im not an expert maybe there is a more formulaic way of doing this, but none of the presentations referenced seem to give it. [Ryd92] gives the closest thing by noting that the different fields are in two groups and the Dirac field is a paradigm of one case and the QED field is the paradigm of the other. I guess if I really wanted to understand this whole thing the QED quantisation (representing gauge theories more generally) would be the place to go.

4 Dyson Series

This is the last peice of the puzzle that describes a correspondence between the quantized Lagrangian and the Feynman diagrams. Im tired of this so Ill be brief. Consider a quantum system with a Hamiltonian of the form

$$H = H_0 + H_I$$

where we understand H_0 and want to understand what the other term produces. Then we can define

$$V(t) = \exp(it(H_0 - H))$$

which it is equivalent to understand. The Dyson series for $V(t)$ is then some formal non-sense that expresses $V(t) = I + \sum_n V_n(t)$ where

$$V_n(t) = \frac{1}{i^n} \int_0^t \int_0^{\tau_n} \cdots \int_0^{\tau_2} H_I(\tau_n) \cdots H_I(\tau_1) \mathrm{d}\tau$$

In scattering problems the quantity of interest is the "scattering operator" or the S-matrix. This can be defined as $S = V(\pm\infty)$, that is substitute the terminals in all the integrals of $V(t)$ to be over all of time. If any of these series converge etc then in principle this integral should return an operator and the thing of interest is how this operator acts on a basis say of our Fock space. The point is that there is something to calculate here. Now one last thing is that it turns out the the integrands can be written as a product of a series of "contractions" of other operators, this is more or less just a canonical way of arranging the operators in a nice way. The point of this is that these contractions are how we construct Feynman diagrams from a Dyson series, roughly the rules are

- For each operator create a vertex
- For each contraction draw a line between the two verticies
- The line direction is determined by the relation between the creation and anihilation operators

5 Feynman Calculus

This is the thing that is actually practically useful so I will be terse. In an experimental context the two things that are of interest are particle decay rates and scattering. For decay rates we care about "the probability per unit time that a particle of a given type will decay", Γ, because particles are all identical this cannot depend on the past of the particle. The decay rate is then the probability for a Poisson process, and we can use that theory to get that the mean lifetime is just the reciprocal of Γ. On the other hand for scattering interactions the relevant quantity is the cross sectional area of the particle (essentially the likely hood of a something else hitting this particle in a bounded region). Because particles will deflect in a smooth region of a particle even defining a cross section, σ, is sticky, it will depend on the type of the two interacting particles, their velocities, charge etc. The precise definition is basically empirical then, being modelled on how we expect the particles to interact in a high energy collision (elastically etc).

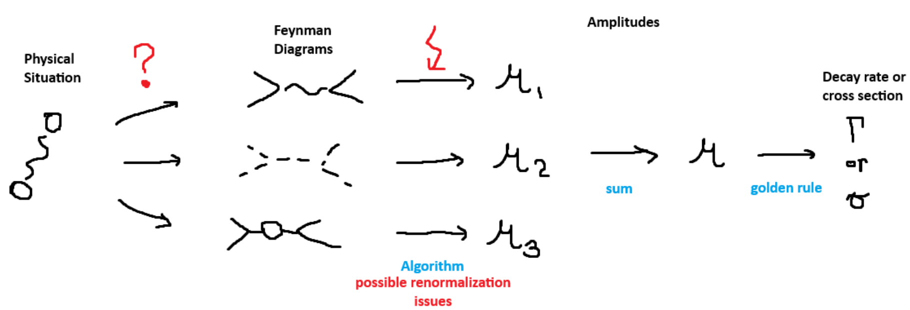
These two quantities are related to the so called "amplitude" or "matrix element", $\mathscr{M}$. Once we have this the theory of QFT produces formulas relating $\mathscr{M}$ to decay rates and cross sections of particles, in the form of "The Golden rule" (im simplifying it because I dont care about the details), let a particle 1 decay into $2, 3, 4, ..., n$ with respective momenta given by p_i then

$$\Gamma = \int |\mathscr{M}(p_1, ..., p_n)|^2 \delta(p_1 - p_2 - \cdots - p_n) \times \prod_{j=2}^{n} \delta(p_j^2 - m_j^2 c^2) \chi_{p_j^0 \geq 0} \quad \mathrm{d}^4 p_j$$

The deltas and chi's should be read as restricting the integral to the regions of the domain where those criteria are satisfied, as they are just "zero everywhere else" and then when the condition is satisfied can be ignored "because they integrate to one" (all delta functions are centered at zero in Griffiths). In particular the deltas in the product are acting on the differential itself (this leads to Griffiths equation 6.21). The golden rule for scattering varies only slightly

$$\Gamma = \frac{1}{\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |\mathscr{M}(p_1, ..., p_n)|^2 \delta(p_1 + p_2 - \cdots - p_n) \times \prod_{j=3}^{n} \delta(p_j^2 - m_j^2 c^2) \chi_{p_j^0 \geq 0} \quad \mathrm{d}^4 p_j.$$

The rest of the game then is determining $\mathscr{M}$ for the specific physical interaction. Griffiths gives a 6 step process for converting a Feynman diagram into an amplitude. The rules are something like assign to edges certain functions of mass and momentum, assign to each verted a delta function of some form and finally integrate over these functions to produce $\mathscr{M}$. It is this integral that has the divergence problems. The process of "renormalizing" seems to be a quasi-systematic way of ignoring this fact. Its probably given in a little more detail in the section 7 examples if I care. Then for a given interaction there will be some number of Feynman diagrams (Not explained) and we take the sum of their amplitudes as the final amplitude in our golden rule.



Remark. In his golden rule Griffiths uses $\delta^4(pi - ...)$, this is because the p_i are 4-vectors and they need to denote this. One can consider the delta function as a product of the delta functions on each components.

Remark. There are lots of other things in this world to see I guess, but I think that's the gist. Something that vaguely interests me is so called "path integral formulation".

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